

International School of Economics at TSU
Econometrics 2
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Problem Set 1 - Review of Probability and Statistics

Instructions: You are encouraged to solve the problems before the recitation. Additionally, you are encouraged to work in groups. It is **not mandatory** to submit solutions unless stated otherwise. However, if you would like to share your solution, I would be happy to review it.

Problem 1 Consider a sample space Ω comprising four possible outcomes: $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$.

Consider also the three events E , F , and G defined as follows:

$$E = \{\omega_1\}, \quad F = \{\omega_1, \omega_2\}, \quad G = \{\omega_1, \omega_2, \omega_3\}$$

Suppose the probabilities of the events are given as:

$$P(E) = \frac{1}{10}, \quad P(F) = \frac{5}{10}, \quad P(G) = \frac{7}{10}$$

Now, consider a fourth event H defined as: $H = \{\omega_2, \omega_4\}$. Find $P(H)$.

Problem 2 Let E and F be two events. Let E^c be a zero-probability event and $P(F) = \frac{1}{2}$. Compute $P(E \cap F)$.

Problem 3 A firm undertakes two projects, A and B. The probabilities of having a successful outcome are $\frac{2}{3}$ for project A and $\frac{4}{5}$ for project B. What is the probability that neither of the two projects will have a successful outcome if their outcomes are independent?

Problem 4 An economics consulting firm has created a model to predict recessions. The model predicts a recession with probability 80% when a recession is indeed coming and with probability 10% when no recession is coming. The unconditional probability of falling into a recession is 20%. If the model predicts a recession, what is the probability that a recession will indeed come?

Problem 5 Let X be a discrete random variable. Let its support R_X be

$$R_X = \{0, 1, 2, 3\}.$$

Let its probability mass function $p_X(x)$ be

$$p_X(x) = \begin{cases} \binom{3}{x} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{3-x} & \text{if } x \in R_X, \\ 0 & \text{if } x \notin R_X, \end{cases}$$

where

$$\binom{3}{x} = \frac{3!}{x!(3-x)!}$$

is a binomial coefficient.

Calculate the probability

$$P(X < 3).$$

Problem 6 Let X be an absolutely continuous random variable. Let its support R_X be

$$R_X = [0, 1].$$

Let its probability density function $f_X(x)$ be

$$f_X(x) = \begin{cases} 1 & \text{if } x \in R_X, \\ 0 & \text{if } x \notin R_X. \end{cases}$$

Compute

$$P\left(\frac{1}{2} \leq X \leq 2\right).$$

Problem 7 Let X be an absolutely continuous random variable with uniform distribution on the interval $[1, 3]$.

Its support is

$$R_X = [1, 3]$$

Its probability density function is

$$f_X(x) = \begin{cases} \frac{1}{2} & \text{if } x \in [1, 3], \\ 0 & \text{otherwise} \end{cases}$$

Compute the expected value of X .

Problem 8 Let X and Y be two random variables, having expected values

$$\mathbb{E}[X] = \sqrt{2}$$

$$\mathbb{E}[Y] = 1$$

Compute the expected value of the random variable Z defined as follows:

$$Z = \sqrt{2}X + Y$$

Problem 9 Let X be a discrete random variable with support

$$R_X = \{0, 1, 2, 3\}$$

and probability mass function

$$p_X(x) = \begin{cases} \frac{1}{4} & \text{if } x \in R_X, \\ 0 & \text{otherwise.} \end{cases}$$

Compute its variance.

Problem 10 Let X be an absolutely continuous random variable with support

$$R_X = [0, 1]$$

and probability density function

$$f_X(x) = \begin{cases} 3x^2 & \text{if } x \in [0, 1], \\ 0 & \text{otherwise.} \end{cases}$$

Compute its variance.

Problem 11 Let X and Y be two random variables such that

$$\text{Var}[X] = 2$$

$$\text{Cov}[X, Y] = 1$$

Compute the covariance

$$\text{Cov}[5X, 2X + 3Y].$$

Problem 12 Let $[X \ Y]$ be a random vector with support

$$R_{XY} = \{[2 \ 2], [2 \ 0], [1 \ 2], [0 \ 2]\}$$

and joint probability mass function

$$p_{XY}(x, y) = \begin{cases} \frac{1}{4} & \text{if } x = 2 \text{ and } y = 2 \\ \frac{1}{4} & \text{if } x = 2 \text{ and } y = 0 \\ \frac{1}{4} & \text{if } x = 1 \text{ and } y = 2 \\ \frac{1}{4} & \text{if } x = 0 \text{ and } y = 2 \\ 0 & \text{otherwise} \end{cases}$$

What is the conditional expectation of X given $Y = 2$?