Econometrics II

Problem Set 3

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Instructions: You are encouraged to solve the problems before the recitation. Additionally, you are encouraged to work in groups. It is not mandatory to submit solutions unless stated otherwise. However, if you would like to share your solution, I would be happy to review it.

Problem 1: The data $\{y_i, x_i, w_i\}$ is from a random sample, $i = 1, \dots, n$. The parameter β is estimated by minimizing the criterion function:

$$S(\beta) = \sum_{i=1}^{n} w_i (y_i - x_i'\beta)^2$$

That is,

$$\hat{\beta} = \arg\min_{\beta} S(\beta)$$

(a) Find an explicit expression for $\hat{\beta}$.

(b) What population parameter is $\hat{\beta}$ estimating?

Be explicit about any assumptions you need to impose, but don't make more assumptions than necessary.

(c) Find the probability limit for $\hat{\beta}$ as $n \to \infty$.

(d) Find the asymptotic distribution of $\sqrt{n}(\hat{\beta} - \beta)$ as $n \to \infty$.

Problem 2: Take the linear equation $Y = Z\beta + e$, and consider the following estimators of β:

(a) $\hat{\beta}_1$: 2SLS using the instruments X_1

(b) $\hat{\beta}_2$: 2SLS using the instruments X_2

(c) $\tilde{\beta}$: GMM using the instruments $X = (X_1, X_2)$ and the weight matrix

$$W = \begin{pmatrix} (X_1'X_1)^{-1}\lambda & 0 \\ 0 & (X_2'X_2)^{-1}(1-\lambda) \end{pmatrix}$$

for $\lambda \in (0, 1)$.

Find an expression for $\tilde{\beta}$ which shows that it is a specific weighted average of $\hat{\beta}_1$ and $\hat{\beta}_2$.

Problem 3: The observations are iid, $(y_i, x_i, q_i : i = 1, \dots, n)$, where x_i is $k \times 1$ and q_i is $m \times 1$. The model is

$$y_i = x_i'\beta + e_i$$
$$\mathbb{E}(x_i e_i) = 0$$
$$\mathbb{E}(q_i e_i) = 0$$

Find the efficient GMM estimator for β.

Problem 4: The model is

$$y_i = z_i\beta + x_i\gamma + e_i$$
$$\mathbb{E}(e_i \mid x_i) = 0$$

Thus z_i is potentially endogenous and x_i is exogenous. Assume that $z_i \in \mathbb{R}$ and $x_i \in \mathbb{R}$.

Someone suggests estimating (β, γ) by GMM, using the pair (x_i, x_i^2) as the instruments. Is this feasible? Under what conditions, if any, (in addition to those described above) is this a valid estimator?

Problem 5: Consider the model

$$y_i = x_i'\beta + e_i$$
$$\mathbb{E}(e_i \mid z_i) = 0$$

with y_i scalar and x_i and z_i each a k vector. You have a random sample $(y_i, x_i, z_i : i = 1, \dots, n)$.

(a) Write the IV estimator $\hat{\beta}$ for β

(b) Suppose that x_i is exogenous in the sense that $\mathbb{E}(e_i \mid z_i, x_i) = 0$. Is $\hat{\beta}$ unbiased for β?

(c) Continuing to assume that x_i is exogenous, find the variance matrix for $\hat{\beta}$, $\text{var}(\hat{\beta} \mid X, Z)$.