International School of Economics at TSU
Microeconomics IV (Game Theory)
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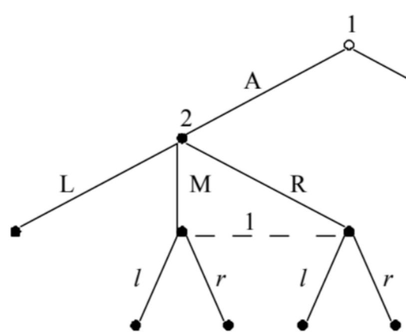
Problem Set 4 - Equilibria of Extensive-Form Games

Instructions: You are encouraged to solve the problems before the recitation. Additionally, you are encouraged to work in groups. It is **not mandatory** to submit solutions unless stated otherwise. However, if you would like to share your solution, I would be happy to review it.

Problem 1 Consider the following extensive-form game:



a. Let σ_1^* denote player 1's mixed strategy in which she plays (B, r) with probability 0.4, (B, l) with probability 0.1, (A, r) with probability 0.3, and (A, l) with probability 0.2. Find the behavioral strategy of player 1 that is equivalent to σ_1^*. Can you find other mixed strategies that are equivalent to σ_1^*?

b. Compute a behavioral strategy of player 1 that is equivalent to her mixed strategy in which she plays (B, r) with probability 0.7, (B, l) with probability 0.3. Is the behavioral strategy uniquely defined? Explain.

Problem 2 Consider the following game. Player 1 first decides between two games A or B, then the game chosen by player 1 is played simultaneously. Player 1 knows which game is played, but player 2 does not know.

Game A

	L	R
U	(1, 0)	(0, 1)
D	(-1, 2)	(2, 0)

Game B

	L	R
U	(4, 3)	(-3, 2)
D	(1, 0)	(0, 0)

a. Find an arbitrary mixed strategy Nash equilibrium of this game in which each pure strategy is played with positive probability.

b. For the mixed strategy equilibrium you obtained in part (a), give an equivalent presentation in behavioral strategies.

Problem 3 Consider the following three-player sequential game. In the first stage, P1 can either play L, ending the game with payoffs $(6, 0, 6)$, or play R, which gives the move to P2. P2 can then play either r, ending the game with payoffs $(8, 6, 8)$, or play ℓ, thereby moving the game to the third stage. In stage 3, P1 and P3 (but not P2) play a simultaneous-move coordination game: they each choose F or G. If their choices differ, they each receive 7 and P2 gets 10; if their choices coincide, all three players get 0.

a. Draw the extensive form game in question.

b. Find ALL subgame perfect equilibria in this game.

c. Take note of P1's choice in stage 1 in all of the subgame perfect equilibria. Discuss carefully whether it could be rational for P1 to choose a different action in stage 1.

Problem 4 Consider the following three-player normal form game, where $S_1 = \{U, D\}$, $S_2 = \{L, R\}$, and $S_3 = \{X, Y\}$:

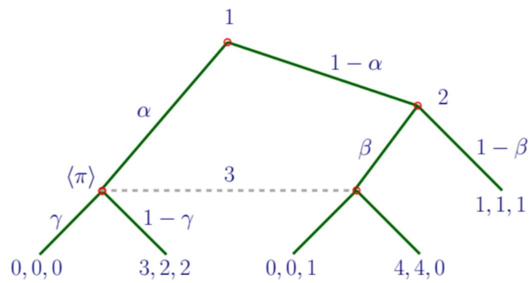
X	L	R
U	$(1, 1, 1)$	$(1, 0, 1)$
D	$(1, 1, 1)$	$(0, 0, 1)$

Y	L	R
U	$(1, 1, 0)$	$(0, 0, 0)$
D	$(0, 1, 0)$	$(1, 0, 0)$

a. Perform iterated deletion of weakly dominated strategies. What are the strategies that survive this process?

b. Find all Nash equilibria (both pure and mixed). Does any of the Nash equilibrium involve weakly dominated strategies?

Problem 5 Find the sequential equilibria of the three-player game represented below. Actions are always left or right. For simplicity, we write behavioral probabilities:



- Let α, β, and γ denote the probabilities with which players 1, 2, and 3 play **Left** at their respective information sets.
- Let π denote player 3's belief that they are at the **left** node of their information set.

We are looking for all combinations of strategies (α, β, γ) and belief π that are **sequentially rational** and **consistent**.