International School of Economics at TSU
Microeconomics IV (Game Theory)
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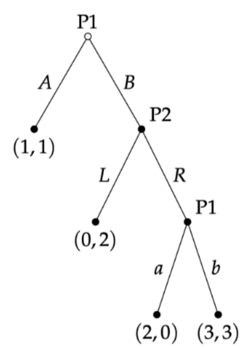
Problem Set 5 - Perfect and Proper Equilibrium. Bayesian Nash Equilibrium. Perfect Bayesian Nash Equilibrium.

Instructions: You are encouraged to solve the problems before the recitation. Additionally, you are encouraged to work in groups. It is **not mandatory** to submit solutions unless stated otherwise. However, if you would like to share your solution, I would be happy to review it.

Problem 1 Consider the following extensive form game



a. Find all subgame perfect equilibria of this game.

b. Convert the game to its normal form. Find all Nash equilibria in pure strategies.

c. Verify whether these Nash equilibria are trembling hand perfect.

Problem 2 Consider the following game:

	L	R
T	$3,1$	$0,0$
M	$0,0$	$1,5$
B	$2,2$	$2,2$

a. Find the perfect, strictly perfect[1], and proper equilibria of this game.

[1]A **strictly perfect equilibrium** is a trembling-hand perfect equilibrium in which each player's strategy is the *unique* best response to the trembled strategies of the opponents. That is, no other strategy yields the same expected payoff, even in the limit of vanishing trembles.

b. Now add a fourth pure strategy for player 1 to this game, whose payoff is equivalent to that of a mixture that places probability 1/4 on T and 3/4 on B.

Notice that in a sense, we are adding nothing new to the game, since player 1 could already achieve the payoffs provided by this strategy by playing the appropriate mixture.

Find the Nash, perfect, strictly perfect, and proper equilibria of this new game.

How do you interpret your results? For example, in your view, is the newly added strategy redundant, or does it add something new to the game?

If redundant, how do you interpret its effect? If not redundant, what new does it add to the game?

In particular, if it is not redundant, how do we know when a model of this sort should include this strategy (and, presumably, many other strategies of this type) and when it should not?

Problem 3 Consider the following Bayesian game.

- Nature selects Game 1 with probability 1/3, Game 2 with probability 1/3, and Game 3 with probability 1/3.
- Player I learns whether Nature has selected Game 1 or not; Player II learns whether Nature has selected Game 2 or not.
- Players I and II simultaneously choose their actions: player I either T or B, and player II either L or R.
- Payoffs are given by the game selected by Nature.

Game 1

	L	R
T	(0,0)	(6,-1)
B	(-1,6)	(4,4)

Game 2

	L	R
T	(1,3)	(0,0)
B	(0,0)	(3,1)

Game 3

	L	R
T	(2,-2)	(-2,2)
B	(-2,2)	(2,-2)

All of this is common knowledge. **Find all the Bayesian Nash equilibria.**

Problem 4 Consider the following Bayesian game. Player 1's type t_1 is drawn from a uniform distribution on the interval from 0 to 1, and payoffs (u_1, u_2) depend on Player 1's type as follows, where ε is a number between 0 and 1 (say $\varepsilon = 0.1$):

	L	R
T	$(\varepsilon t_1,\, 0)$	$(\varepsilon t_1,\, -1)$
B	$(1,\, 0)$	$(-1,\, 3)$

Derive the Bayesian Nash equilibrium of the given game.

Perfect Bayesian Equilibrium (PBE)

A perfect Bayesian equilibrium consists of strategies and beliefs satisfying Requirements 1 through 4:

- **Requirement 1:** At each information set, the player with the move must have a belief about which node in the information set has been reached by the play of the game.
- **Requirement 2:** Given their beliefs, the players' strategies must be sequentially rational.
- **Requirement 3:** At information sets on the equilibrium path, beliefs are determined by Bayes' rule and the players' equilibrium strategies.
- **Requirement 4:** At information sets off the equilibrium path, beliefs are determined by Bayes' rule and the players' equilibrium strategies where possible.

For dynamic games, we have

$$\text{PBE} \subseteq \text{SPE} \subseteq \text{NE},$$

which gives us a standard method to find perfect Bayesian equilibrium.

Problem 5 In the following extensive-form games, derive the normal-form game and **find all the pure-strategy Nash, subgame-perfect, and perfect Bayesian equilibria.**

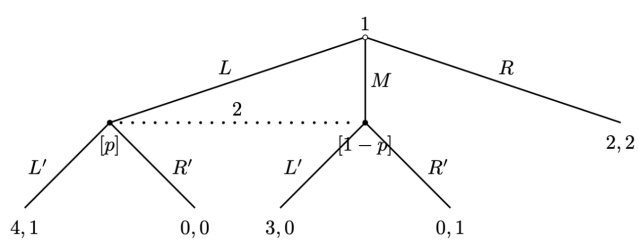


Figure 1: Game 1

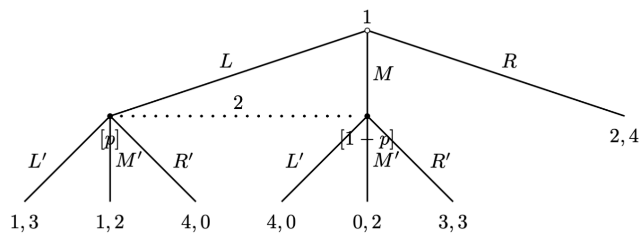


Figure 2: Game 2

Problem 6 Find all perfect Bayesian equilibria in the following signaling game.

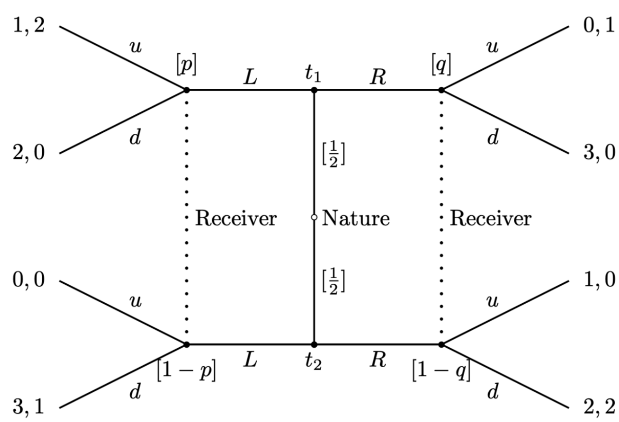


Figure 3: Game 3

Problem 7 A buyer and a seller have valuations v_b and v_s. It is common knowledge that there are gains from trade (i.e., that $v_b > v_s$), but the size of the gains is private information, as follows: the seller's valuation is uniformly distributed on $[0, 1]$; the buyer's valuation is $v_b = k \cdot v_s$, where $k > 1$ is common knowledge; the seller knows v_s (and hence v_b) but the buyer does not know v_b (or v_s). Suppose the buyer makes a single offer, p, which the seller either accepts or rejects. What is the perfect Bayesian equilibrium when $k < 2$? When $k > 2$?